Closing today: 4.1(1) and 4.1(2)
Closing Wed: 4.3
Closing Fri: 4.4

4.3 Classifying Critical Points

$\boldsymbol{y = f(x)}$	$\boldsymbol{y' = f'(x)}$
horiz. tangent	zero
increasing	positive
decreasing	negative
vertical tangent, sharp corner, or not continuous	does not exist

Key, big, essential observation

Let $y = f(x)$ have a critical number at $x = a$; if $f'(x)$ changes from…

1. …positive to negative, then a **local maximum** occurs at $x = a$.
2. …negative to positive, then a **local minimum** occurs at $x = a$.

This is called the first derivative test.

Example:

1. Find and classify the critical numbers for

$$y = x^3 + 3x^2 - 72x$$

2. Find and classify the critical numbers of

$$y = x^4 - 2x^3$$

3. Find and classify the critical numbers of

$$y = x^{2/3}$$

4. Find and classify the critical numbers of

$$y = \frac{x^3}{x^2 - 1}$$

The 2nd Derivative

$$y'' = f''(x) = \frac{d}{dx}(f'(x))$$

= "rate of change of first derivative"

Terminology

If **f''(x) is positive**,
then the **slope of f(x) is increasing** and
we say f(x) is **concave up**.

If **f''(x) is negative**,
then the **slope of f(x) is decreasing** and
we say f(x) is **concave down**.

A point in the domain of the function
at which the concavity changes is
called an **inflection point**.

Example: Find all inflection points of

$$y = x^4 - 2x^3$$

Summary:

$\boldsymbol{y = f(x)}$	$\boldsymbol{y'' = f''(x)}$
possible inflection	zero
concave up	Positive
concave down	Negative
possible inflection	does not exist

Big Observation:

If a graph is **concave up at a critical number**, then it is a **local minimum**.

If a graph is **concave down at a critical number**, then it is a **local maximum**.

This is called the 2nd derivative test.

Example: Find all critical numbers of

$$y = 2 + 2x^2 - x^4$$

and classify them using the 2nd derivative test.